Microeconomics Theory I

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(TeIAS)

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Introduction & Objectives

The course covers:

- Decision theoretic models of choice: with and without uncertainty
- Classical theories of consumer and producer behavior
- Classical theories of markets and the competitive equilibrium

The objective of the course is to provide students with a firm grounding in the analytic methods of microeconomics theory.

Although the course is centered around classical theories of economic behavior, new developments in these fields will also be discussed.

General Information

Email: sepehrekbatani@teias.institute

Lectures: Sat, Mon 11AM-12:30PM

Office Hours: Sat 3-4PM

Textbooks:

- Riley, Essential Microeconomics, Cambridge University Press, 2012
- [Supplementary] Mas-Colell, Whinston, Green, Microeconomics Theory, Oxford University Press, 1995.

Course Outline

1. Introduction and Maximization:
   Riley, Chapter 1
2. Decision Theory & Consumer Theory:
   Riley, Chapter 2
3. Producer Theory:
   Riley, Chapter 4
4. Markets and Equilibrium:
   Riley, Chapters 3 and 5
5. Choice under Uncertainty:
   Riley, Chapter 7
6. Externalities and Public Goods:
   MWG, Chapter 11
7. Equilibrium in Financial Markets:
   Riley, Chapter 8

Teaching Assistants

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  - OH: Tue 9:30-10:30 AM

Section 1

Maximization

Example 1 - Profit Maximizing Firm

Cost Function: $C(q) = 5 + 12q + 3q^2$

Demand price function: $p(q) = 20 - q$

Solve for the profit maximizing output and price?

Example 2 - Two Products

Model 1

Cost function: $C(q) = 10q_1 + 15q_2 + 2q_1^2 + 3q_1q_2 + 2q_2^2$

Demand price function: $p_1 = 85 - \frac{1}{4}q_1$ and $p_2 = 90 - \frac{1}{4}q_2$

Solve for the profit maximizing outputs?

Model 2

Cost function: $C(q) = 10q_1 + 15q_2 + q_1^2 + 3q_1q_2 + q_2^2$

Demand price function: $p_1 = 65 - \frac{1}{4}q_1$ and $p_2 = 70 - \frac{1}{4}q_2$

Solve for the profit maximizing outputs?

Model 1

Revenue:

$$R_1 = p_1 q_1 = (85 - \frac{1}{4} q_1) q_1 = 85 q_1 - \frac{1}{4} q_1^2$$

$$R_2 = p_2 q_2 = (90 - \frac{1}{4} q_2) q_2 = 90 q_2 - \frac{1}{4} q_2^2$$

Profit:

$$\begin{aligned}\pi &= R_1 + R_2 - C \\ &= 85 q_1 - \frac{1}{4} q_1^2 + 90 q_2 - \frac{1}{4} q_2^2 - (10 q_1 + 15 q_2 + 2 q_1^2 + 3 q_1 q_2 + 2 q_2^2) \\ &= 75 q_1 + 75 q_2 - \frac{9}{4} q_1^2 - \frac{9}{4} q_2^2 - 3 q_1 q_2\end{aligned}$$

Think on the Margin

Marginal profit of increasing q_1:

$$\frac{\partial \pi}{\partial q_1} = 75 - \frac{9}{2}q_1 - 3q_2.$$

Therefore the profit maximizing choice is:

$$q_1 = m_1(q_2) = \frac{2}{9}(75 - 3q_2) = \frac{2}{3}(25 - q_2)$$

Marginal profit of increasing q_2:

$$\frac{\partial \pi}{\partial q_2} = 75 - 3q_1 - \frac{9}{2}q_2.$$

Therefore the profit maximizing choice is:

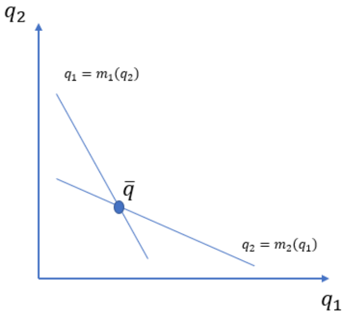
$$q_2 = m_2(q_1) = \frac{2}{9}(75 - 3q_1) = \frac{2}{3}(25 - q_1)$$

Profit Maximizing Outputs

$$q_1 = m_1(q_2) = \frac{2}{3}(25 - q_2),\ q_2 = m_2(q_1) = \frac{2}{3}(25 - q_1)$$

If you solve for q satisfying both equations you will find that the unique solution is:

$$\bar{q} = (\bar{q_1}, \bar{q_2}) = (10, 10)$$



Model 2

Cost function: $C(q) = 10q_1 + 15q_2 + q_1^2 + 3q_1q_2 + q_2^2$

Demand functions: $p_1 = 65 - \frac{1}{4}q_1$ and $p_2 = 70 - \frac{1}{4}q_2$

Revenue:

$$R_1 = p_1q_1 = (65 - \frac{1}{4}q_1)q_1 = 65q_1 - \frac{1}{4}q_1^2$$

$$R_2 = p_2q_2 = (70 - \frac{1}{4}q_2)q_2 = 70q_2 - \frac{1}{4}q_2^2$$

Profit:

$$\begin{aligned}\pi &= R_1 + R_2 - C \\ &= 55q_1 + 55q_2 - \frac{5}{4}q_1^2 - \frac{5}{4}q_2^2 - 3q_1q_2\end{aligned}$$

Think on the Margin

Marginal profit of increasing q_1:

$$\frac{\partial \pi}{\partial q_1} = 55 - \frac{5}{2}q_1 - 3q_2.$$

Therefore for any q_2 profit maximizing q_1 is:

$$q_1 = m_1(q_2) = \frac{2}{5}(55 - 3q_2)$$

Marginal profit of increasing q_2:

$$\frac{\partial \pi}{\partial q_2} = 55 - 3q_1 - \frac{5}{2}q_2.$$

Therefore for any q_1 profit maximizing q_2 is:

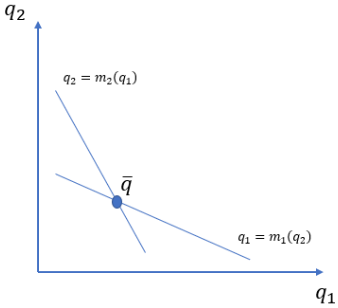
$$q_2 = m_2(q_1) = \frac{2}{5}(55 - 3q_1)$$

Profit Maximizing Outputs

$$q_1 = m_1(q_2) = \frac{2}{5}(55 - 3q_2),\ q_2 = m_2(q_1) = \frac{2}{5}(55 - 3q_1)$$

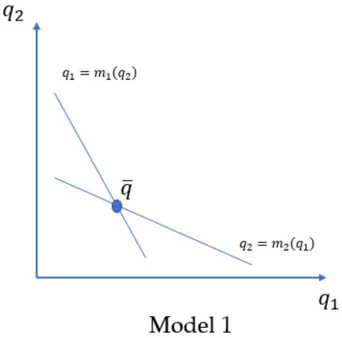
If you solve for $\bar{q}$ satisfying both equations you will find that the unique solution is:

$$\bar{q} = (\bar{q_1}, \bar{q_2}) = (10, 10)$$



Comparison Between Two Models

These look very similar to the profit-maximizing lines in Model 1. However, now the profit-maximizing line for q_2 is steeper (i.e. has a more negative slope).



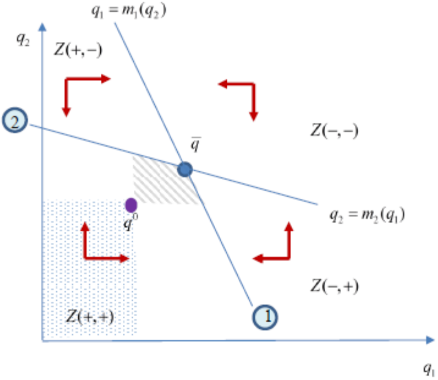
As we shall see this makes a critical difference.

Model 1 Analysis

Is $\bar{q} = (\bar{q_1}, \bar{q_2})$ the profit maximizing output?

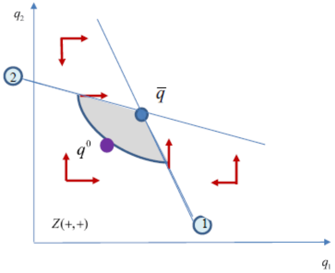
The arrows indicate the directions in which $\pi(q_1, q_2)$ increases.

Consider the point q^0. Profit is higher in the shaded region and lower in the dotted region. The level set through q^0 must have a negative slope.



Model 1 Analysis Cont.

The level set $\pi(q) = \pi(q^0)$ in the $Z(+,+)$ region.

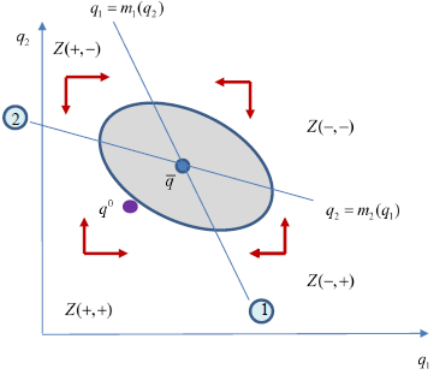


Profit is higher in the shaded region.

Note that the level set is parallel to the q_2 axis at the point of intersection with the maximizing line for q_2 and is parallel to the horizontal axis at the point of intersection with the maximizing line for q_1.

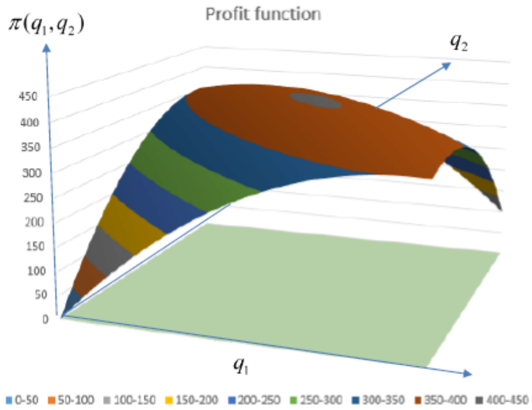
Model 1 Analysis Cont.

The level set $\pi(q) = \pi(q^0)$ and superlevel set $\pi(q) \geq \pi(q^0)$:



Model 1 Analysis Cont.

The profit, $\pi(q_1, q_2)$ is depicted below:

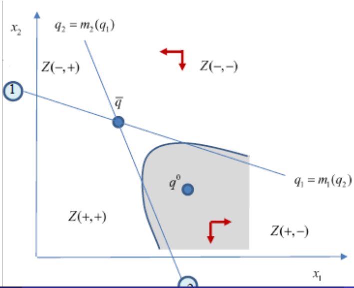


Model 2 Analysis

There are four Zones:

- $Z(+,+)$: The zone in which profit is increasing in q_1 and q_2.
- $Z(+,-)$: The zone in which profit is increasing in q_1 and decreasing in q_2.
- ...

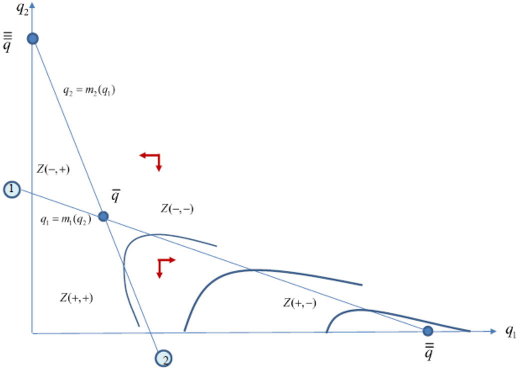
Never leads to the intersection point $\bar{q} = (10, 10)$



Model 2 Analysis Cont.

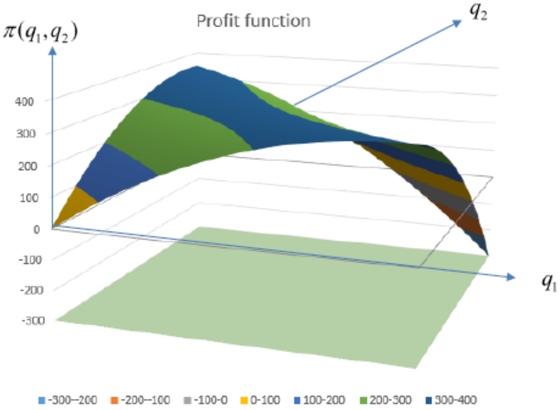
Local Maximum $\bar{\bar{q}} = (\bar{\bar{q}}_1, 0)$ on the q_1 axis.

By an essentially identical argument, there is a second local maximum $\bar{\bar{\bar{q}}}$ on the q_2 axis.



Model 2 Analysis Cont.

The profit function has the shape of a saddle. The output vector $\bar{q}$ where the slope in the direction of each axis is zero is called a saddle-point.



Interior and Boundary Solution

Model 1 has an **Interior Solution**.

Model 2 has two **Boundary Solutions**.

When solving for such an example, remember to check for both types of solutions and compare the value of the function you are maximizing.

First Order Condition

Consider the two variable problem:

$$\max_{q} f(q_1, q_2)$$

First Order Necessary Conditions for a Maximum

For $\bar{q} \gg 0$ to be a maximizer, the following two conditions must hold:

$$\frac{\partial f}{\partial q_1}(\bar{q}) = 0 \quad \text{and} \quad \frac{\partial f}{\partial q_2}(\bar{q}) = 0$$

Second Order Condition

Suppose that the first order necessary conditions hold at $\bar{q}$. A necessary condition for a maximum is that the slope must be decreasing.

Second Order Necessary Conditions for a Maximum

For $\bar{q} \gg 0$ to be a maximizer, the following two conditions must hold:

$$\frac{\partial}{\partial q_1}\frac{\partial f}{\partial q_1}(\bar{q_1}, \bar{q_2}) \leq 0 \quad \text{and} \quad \frac{\partial}{\partial q_2}\frac{\partial f}{\partial q_2}(\bar{q_1}, \bar{q_2}) \leq 0$$

Sufficient Conditions

As we have seen, these conditions are necesaary for a maximum, by they **don't** guarantee that $\bar{q}$ is the maximum.

Sufficient Conditions for a Local Maximum

If the first and second order necessary conditions hold at $\bar{q}$ and the level sets are closed loops around $\bar{q}$, then the function $f(q)$ has a local maximum at $\bar{q}$.

Sufficient Conditions for a Global Maximum

If the first and second order necessary conditions hold at $\bar{q}$ and the level sets are closed loops around $\bar{q}$ and the FOC hold only at $\bar{q}$, then this is the global maximizer.

Non-Negativity Constraints

Many economic variables cannot be negative. Suppose this is true for all variables

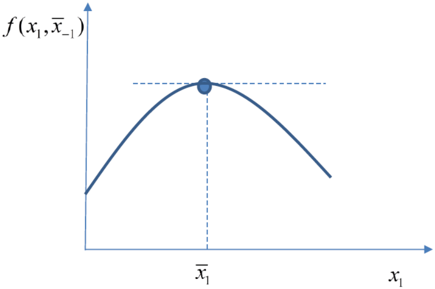
Let $\bar{x} = (\bar{x_1}, ..., \bar{x_n})$ solve $\max_{x \geq 0}\{f(x)\}$.

We will consider the first variable. It is helpful to write the optimal value of all the other variables as $\bar{x}_{-1}$. Then:

$$\bar{x}_1 \text{ solves } max_{x_1 \geq 0} f(x_1, \bar{x}_{-1}).$$

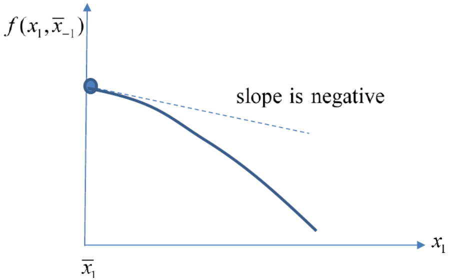
Case I: $\bar{x}_1 > 0$

For x_1 to be the maximizer, the slope of $f(x_1, \bar{x}_{-1})$ must be zero at x_1.



Case II: $\bar{x}_1 = 0$

For $\bar{x}_1$ to be the maximizer, the slope of $f(x_1, \bar{x}_{-1})$ cannot be strictly positive at $\bar{x}_1$.



Necessary condition can be written as:

$$\frac{\partial f}{\partial x_1}(\bar{x}) \leq 0, \text{ with equality if } \bar{x}_1 > 0$$

Section 2

Constrained Maximization

Constrained Maximization - An Economic Approach

Problem:

$$\max_{x \geq 0} f(x) \text{ subject to } g(x) \leq b$$

Let $\bar{x}$ be the solution to this problem.

Interpretation, if the firm chooses x it requires $g(x)$ units of a resource that is fixed in supply (i.e. floor space of plant, highly skilled workers)

Assumption 1: No solution, $x^* = arg \max_{x \geq 0} f(x)$ satisfies the resource constraint. Therefore, at $\bar{x}$, this constraint is binding.

We interpret $f(x)$ as the profit of the firm.

Shadow Price

To solve this problem, we consider the **relaxed problem** in which the firm can purchase additional units at the price λ. Since this is a hypothetical opportunity cost, economists refer to the price as the **shadow price** of the resource rather than a market price.

Suppose that the firm purchases $g(x) - b$ additional units. Its profit is then:

$$\textbf{Lagrangian}: \ \mathcal{L} = f(x) - \lambda(g(x) - b) = f(x) + \lambda(b - g(x))$$

The relaxed problem is then:

$$\max_{x \geq 0} \mathcal{L} = f(x) + \lambda(b - g(x))$$

Shadow Price Cont.

Suppose that $\bar{x}$ solves the optimization problem. The net gain to increase x_j is:

$$\frac{\partial f}{\partial x_j}(\bar{x}) - \lambda \frac{\partial g}{\partial x_j}(\bar{x})$$

- If the net gain is strictly positive, the firm gains by increasing x_j. Thus:

$$\frac{\partial f}{\partial x_j}(\bar{x}) - \lambda \frac{\partial g}{\partial x_j}(\bar{x}) \leq 0$$

- If the net gain is strictly negative, the firm gains by reducing x_j. Thus:

$$\frac{\partial f}{\partial x_j}(\bar{x}) - \lambda \frac{\partial g}{\partial x_j}(\bar{x}) \geq 0 \text{ when } \bar{x}_j > 0$$

Thus the marginal net gain is zero at the optimum unless the optimum is zero.

Constraint Qualification

Constraint Qualification

Define X to be the set of feasible vectors; that is:

$$X = \{x | x \geq 0 \text{ and } h_i(x) \geq 0, i = 1, ..., m\}$$

The constraint qualification is said to hold at $\bar{x} \in X$ if:

i. For binding constraints at $\bar{x}$, the associated gradient vector $\dfrac{\partial h_i}{\partial x}(\bar{x}) \neq 0$,

ii. $\bar{X}$, the set of non-negative vectors satisfying the linearized binding constraints, has a non-empty interior.

Kuhn-Tucker Conditions

Kuhn-Tucker Conditions (First Order Conditions)

Suppose $\bar{x}$ solves the following problem

$$\max_x \{f(x) | x \in X\} \text{ where } X = \{x | x \geq 0, h_i(x) \geq 0, i = 1, ..., m\}$$

If Constraint Qualification holds at $\bar{x}$, then there exists a vector of shadow prices $\lambda \geq 0$ such that:

$$\frac{\partial \mathcal{L}}{\partial x_j}(\bar{x}, \lambda) \leq 0, j = 1, ..., n \text{ with equality if } \bar{x}_j > 0$$

and

$$\frac{\partial \mathcal{L}}{\partial \lambda}(\bar{x}, \lambda) \geq 0, i = 1, ..., m \text{ with equality if } \lambda_i > 0$$

Necessary and Sufficient Conditions

Necessary and Sufficient Conditions for a Maximum

If f and $h_i, i = 1, ..., m$ are all quasi-concave, the Kuhn-Tucker conditions hold at $\bar{x}$ and for each binding constraint, $\frac{\partial h_i}{\partial x}(\bar{x}) \neq 0$, then $\bar{x}$ solves

$$\max_x \{f(x) | x \geq 0, h_i(x) \geq 0, i = 1, ..., m\}.$$

Concave Function

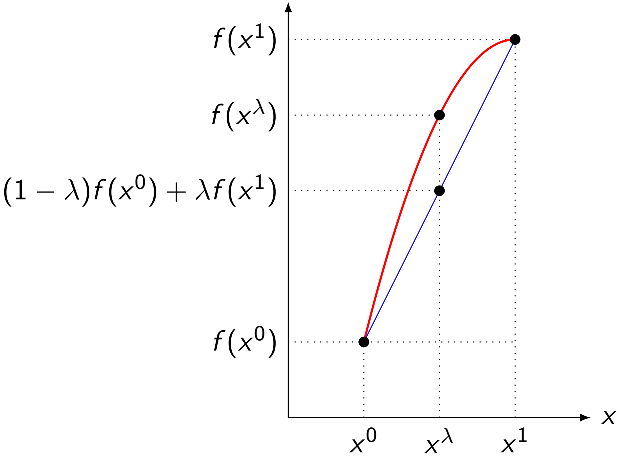
Definition: Concave and Strictly Concave Function

A function f is concave on the interval $[a, b]$ if, for any two points x^0 and x^1 in this interval, and any convex combination $x^\lambda = (1 - \lambda)x^0 + \lambda x^1$, $0 < \lambda < 1$,

$$f(x^\lambda) \geq (1 - \lambda)f(x^0) + \lambda f(x^1)$$

The function is strictly concave if the inequality is always strict.

Concave Function - Example

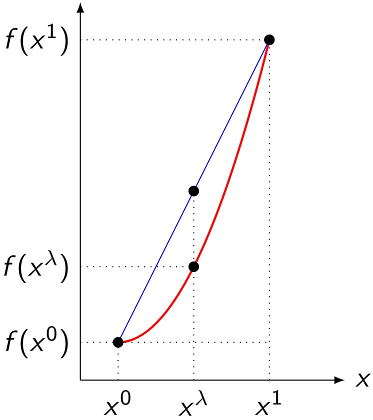


Quasi-Concave Function

Definition: Quasi-Concave Function

A function f is quasi-concave over the interval $[a, b]$ if, for any two points x^0 and x^1 in this interval such that $f(x^1) \geq f(x^0)$, $f(x^\lambda) \geq f(x^0)$ where $x^\lambda = (1 - \lambda)x^0 + \lambda x^1$ and $\lambda \in (0, 1)$.

Quasi-Cocave Function - Example



Example

$$\max_x \{f(x) = \ln(1 + x_1) + \ln(1 + x_2) | x \geq 0, h(x) = 2 - x_1 - x_2 \geq 0\}$$

The Lagrangian for the problem is:

$$\mathcal{L}(x, \lambda) = \ln(1 + x_1) + \ln(1 + x_2) + \lambda(2 - x_1 - x_2)$$

The FOC are as follows:

$$\frac{\partial \mathcal{L}}{\partial x_1} = \frac{1}{1 + x_1} - \lambda = 0,$$
$$\frac{\partial \mathcal{L}}{\partial x_2} = \frac{1}{1 + x_2} - \lambda = 0,$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = 2 - x_1 - x_2 = 0.$$

From the first two conditions $\bar{x}_1 = \bar{x}_2$. Substituting into the constraint, $\bar{x} = (1, 1)$.

Section 3

Envelope Theorem

Effect of Parameter Change on Maximized Value

Consider a multiproduct price taker firm who has a cost function $C(q)$.

Maximized profit is $\Pi(p) = \max_q\{p.q - C(q)\}$. Suppose that for any p there is a unique $q^*(p)$.

With data only on profit and prices and outputs, what can be said about the elasticity of profit with respect to the price of commodity j?

$$\epsilon(\Pi, p_j) = \frac{\partial \Pi}{\partial p_j}\frac{p_j}{\Pi}$$

Example, $\epsilon =?$

$$p^0 = 100, C(q) = q^2$$

Profit, $pq - q^2$ is maximized at $q^*(p) = \frac{1}{2}p = 50$ and so $\Pi^0 = 2500$.

Simplistic answer: Assume that the firm doesn't change its output.

$$\frac{\partial \Pi}{\partial p} = q^*(p^0) = 50 \longrightarrow \epsilon(\Pi, p) = \frac{\partial \Pi}{\partial p}\frac{p}{\Pi} = 50 \times \frac{100}{2500} = 2$$

Sophisticated response: Take into account the effect of output change.

$$\Pi(p) = pq^*(p) - C(q^*) = pq^*(p) - {q^*}^2 = p(\frac{1}{2}p) - (\frac{1}{2}p)^2 = \frac{1}{4}p^2$$

$$\rightarrow \frac{\partial \Pi}{\partial p}(p) = \frac{1}{2}p \text{ and } \epsilon(\Pi, p) = \frac{\partial \Pi}{\partial p}\frac{p}{\Pi} = \frac{1}{2}p\frac{p}{\Pi} = 50 \times \frac{100}{2500} = 2$$

Same answer! What is going on here?

Envelope Theorem

Envelope Theorem

Define $F(\alpha) = f(x^*(\alpha), \alpha)$ where $x^*(\alpha) = \mathit{arg}\max_x\{f(x,\alpha)|x \in X \subset \mathbb{R}^n\}$ and $f \in \mathbb{C}^1$.

If $x^*(\alpha)$ is a continuous function then:

$$\frac{dF}{d\alpha} = \frac{\partial f}{\partial \alpha}(x^*(\alpha), \alpha)$$